Rights and wrongs about wireless network centralized or meshed schemes performance

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Abstract: We consider the problem of pumping megabit per second and per hectar by a wireless network. This challenge is important since the aim of a wireless network is to cover an urban area where the demand of traffic may be dense and widely distributed in space. We show that the centralized schemes have the same disadvantages as distributed meshed schemes, the later being less costly to deploy than the former since there is a minimum density of access point needed. We show that an hybridation of mesh over access points may be the ultimate best approach to wireless networks.

Key-words: wireless mesh networks, routing, access points, performance and models

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Le vrai du faux au sujet des réseaux sans fil centralisés ou maillés

Résumé : Nous nous intéressons au problème du pompage des méga bits par secondes et par hectare au travers d'un réseau sans fil. Ce défi est important car l'objectif d'un réseau sans fil est de couvrir un zone urbaine ou semi-urbaine où la demande de trafic sera dense et répartie dans l'espace. Nous montrons que le schéma centralisé présente les mêmes desavantages que le modèle distribué, ce dernier étant moins couteux au déploiment puisqu'il permet une densité minime de points d'accès. Nous montrons que l'hybridation de ces deux schémas apporte la réponse optimale aux problèmes de performance des réseaux sans fil.

Mots-clés : réseaux sans fil maillés, routage, points d'accès, performances et modèles

1 introduction

Wireless networks are expected to be deployed extensively in densely populated areas, urban or semi-urban areas. There are two main schemes for such deployements:

- the centralized access point scheme;
- the meshed scheme.

Recent experiments showed that with WiFi the performance of central access point is around 5 Mbps and mesh networks is around 0.5 Mbps. Some have concluded that mesh networks perform ten times worse than central access point. This is wrong because the comparison is not made on equal footing: the access point network is isolated while the mesh network is distributed on a two dimension map. We will show that several access points distributed on a two dimension map will perform ten times worse than isolated access point network, therefore performing as well as mesh networks but with more constraints and more infrastructure costs.

The apparent performance drop between meshed scheme and access point scheme is currently taken as an excuse by some manufacturers to not invest in the meshed scheme. We think that this is a mistake. In fact the comparison is between a problem statement in dimension zero (one single access point) with a problem statement in dimension two (a meshed area).

In the case of dimension zero problem statement with a single access point there is no question to pump several megabit per second and per square meter. In order to consider the possibility to extend the network on an area we must consider several access points deployed on an area. If we consider a non zero density of access points on an infinite area, then we have a domension two problem statement. In this case we will show that the interference between the access points and mobile will create the same drop of performance as with meshed networks.

2 Network model for dimension zero wireless network

We take a simplified model of IEEE 802.11 (WiFi). We have a network made of N nodes connected to an access point via WiFi. We consider a slotted time and nodes transmit packet that fit slots. The traffic is Poisson with a mean λ emitter per slot. Therefore the average per node traffic is $\rho = \frac{\lambda}{N}$.

As with slotted Aloha when two nodes or more transmit (to the access point), the result is a collision and no packet will be decyphered. We assume that the slot contain the possibility for the receiver to transmit a short acknowledgement of the emitted packet, so that collision are detected in real time as in IEEE 802.11. Therefore a slot is empty with probability $e^{-\lambda}$, contains a succesful transmission with probability $\lambda e^{-\lambda}$, and contains a collision with probability $1 - e^{-\lambda} - \lambda e^{-\lambda}$. Similarly a packet transmission is succesful with probability $e^{-\lambda}$ (when no other emitter contend on the same slot) and collides with probability $1 - e^{-\lambda}$.

When a packet collide, the emitter randomly schedules a retransmission as in Aloha, or as in IEEE 802.11 with an exponential binary backoff. We assume that the retransmission process does not affect the Poisson nature of the Poisson traffic (that cumulate new packet transmission and old packet packet retransmission). Therefore the average number of (re)transmissions per packet will be e^{λ}.

The maximum net throughput (*i.e.* the throughput excluding retransmissions) is equal to the maximum of function $\lambda e^{-\lambda}$ which is e^{-1} attained for $\lambda = 1$ and is independent of the number of nodes in the network. Notice that with pure CSMA with variable packet length as specified for WiFi we would get a maximum throughput which $1 - O(\frac{1}{\sqrt{L}})$ where L is the average packet length (in slots). Anyhow the spirit of the story is that the maximum throughput is a constant independent of the size of the networks.

3 Network model for dimension two wireless networks

The model is a network made of N nodes on a large map of area $\mathcal{A}$ uniformly distributed with constant density $\nu = \frac{N}{\mathcal{A}}$. We keep the slotted time model. We assume that at each slot the spatial density of transmitters is Poisson with uniform mean λ per square unit. Therefore the average per node traffic is $\rho = \frac{\lambda}{\nu}$.

We assume that the attenuation coefficient is $\alpha > 2$, for example $\alpha = 2.5$: the signal level of a transmission received at distance r is $W = \frac{1}{r^\alpha}$. The question is to give an estimate of the probability that a message can be received correctly by a given receiver. In the two dimension formulation of the problem this probability will depend on the distance to the emitter. It depends also to the minimum Signal over Noise Ratio (SNR) that is acceptable for correctly processing the packet. In other words we assume that the packet is correctly received if its signal is K time larger than the sum of the signal of the other emitters on the same slot. Typically K is of order 4 or 10.

It is to say that if $\mathcal{S}$ is the set of the locations z_i of the nodes transmitting during this slot, and $z_0 \in \mathcal{S}$:

$$z_0 \text{ received by } z \Longleftrightarrow |z - z_0|^{-\alpha} > K \sum_{z_i \in \mathcal{S} - \{z_0\}} |z - z_i|^{-\alpha}$$

or $W(z, \{z_0\}) > KW(z, \mathcal{S} - \{z_0\})$ where $W(z, \mathcal{S}) = \sum_{z_i \in \mathcal{S}} |z - z_i|^{-\alpha}$. Figure 1 shows the function $W(z, \mathcal{S})$ for z varying in the plan with $\mathcal{S}$ an arbitrary random set of transmitter. We take $\alpha = 2.5$. It is clear that the closer the receiver is to the emitter then the larger is the probability to correctly receive the packet. We can draw around each emitter the area where the packet is received correctly. The aim is to find the average size of this area and how it is function of parameters K and λ. This is the aim of the next section. Figure **??** displays areas of correct reception around transmitter for $K = 1, 4, 10$ and $\alpha = 2.5$.

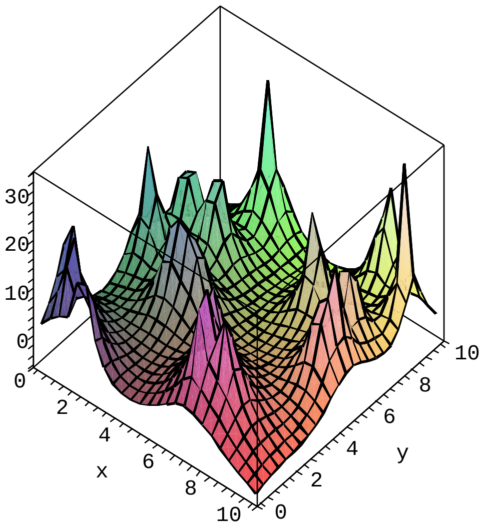

Figure 1: Signal levels landscape (in dB) for a random network with $\alpha = 2.5$.

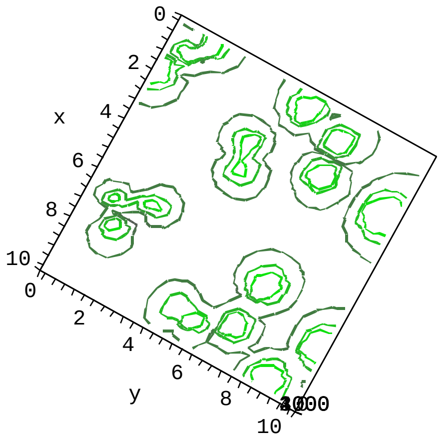

Figure 2: distribution of correct reception areas for a random network with $\alpha = 2.5$ for various SNR parameters $K = 1, 4, 10$.

3.1 Distribution of correct receptions

We know [1, 3] that the Laplace transform of the signal level $W(z, \mathcal{S}(\lambda))$ (assuming all transmitters tuned at one unit nominal power) can be exactly calculated when $\mathcal{S}(\lambda)$ is given by a 2D Poisson process with intensity of λ transmitter per slot and per square area unit. The random variable $W(z, \mathcal{S}(\lambda))$ is invariant by translation and does not depend on z. We denote $W(\lambda) \equiv W(z, \lambda)$.

Theorem 1 *The Laplace transform $w(\theta, \lambda) = E(e^{-W(\lambda)\theta})$:*

$$w(\theta, \lambda) = \exp(-\lambda\pi\Gamma(1 - \frac{2}{\alpha})\theta^{\frac{2}{\alpha}}) \tag{1}$$

If we assume that the signal can be altered by a random fading factor $\exp(F)$ then the factor $\Gamma(1 - \frac{1}{\alpha})$ is to be simply replaced by $\Gamma(1 - \frac{1}{\alpha})E(e^{-\frac{2}{\alpha}F})$. The Laplace transform is still of the form $\exp(-\lambda C\theta^{\gamma})$ with $\gamma = \frac{2}{\alpha}$.

The question now is to know what is the probability that a random receiver receives a correct packet. This probability is given by the product of the traffic density λ and the average area $A(\lambda)$ of correct reception of a random transmission.

Theorem 2 *The probability that a random receiver receives a correct packet on a given slot is equal to σ_1 where*

$$\sigma_1 = \frac{\sin(\frac{2}{\alpha}\pi)}{\frac{2}{\alpha}\pi} K^{-\frac{2}{\alpha}}. \tag{2}$$

Notice that when $\alpha = 2.5$ and $K = 10$ we have $\sigma_1 = 0.037066$ which is *ten* times smaller that we could get with dimension zero wireless networks. Notice that this quantity is independent of λ since the network is supposed to be unbounded.

Proof: The average area $A(\lambda)$ of correct reception is given by the expression $A(\lambda) = 2\pi \int_0^{\infty} p(r, K, \lambda) r dr$ with $p(r, K, \lambda) = P(W(\lambda) < \frac{1}{K} r^{-\alpha})$. Using the fact that $p(r, K, \lambda) = p(r\sqrt{\lambda}, K, 1)$ we get $A(\lambda) = \frac{\sigma_1}{\lambda}$ with

$$\begin{aligned} \sigma_1 &= \frac{e^{i\pi\gamma} - e^{-i\pi\gamma}}{2i} K^{-\gamma}\Gamma(1-\gamma) \int_0^{\infty} \exp(-C\theta^{\gamma})\theta^{\gamma-1} d\theta \\ &= \sin(\pi\gamma) K^{-\gamma} \frac{\Gamma(1-\gamma)}{C\gamma} \end{aligned}$$

therefore $\sigma_1 = \frac{\sin(\pi\gamma)}{\pi\gamma} K^{-\gamma} \frac{\Gamma(1-\gamma)}{C}$. For no fading case we get $\sigma_1 = \frac{\sin(\pi\gamma)}{\pi\gamma} K^{-\gamma}$.

3.2 Consequence on the performance of wireless networks

We have identified that the significant drop in network capacity is the natural consequence of wave propagation and Signal to Noise Ratios in 2D network. This is not the consequence of routing strategy: access points or mesh networking.

Of course, considering that e^{-1}, the maximum capacity in dimension zero, is much larger than the maximum capacity σ_1 in dimension 2, there is the temptation to play a dimension zero protocol on a dimension 2 network, *i.e.* to have a single access point to cover a whole area. For this goal, one should assign a traffic density λ small enough so that the total traffic load $\iint \lambda$ be equal to 1 or at least of order 1. This is definitely not a good idea since it would lead to local traffic exactly equal to zero when the network size area increases.

The main real question is how to pump a significant number of Mbps per hectar in a potentially infinite network map? We will investigate this question on access point schemes and on mesh schemes in the next section.

4 Pumping Megabit per second per hectar

4.1 The access point based schemes

We have seen that a single access point scheme cannot pump a non zero traffic density in an infinite network map. Therefore the appropriate access point scheme consists into dispatching several a constant density ν_a of access points in the plan. Therefore their will be an infinite number of such access points, the mobile user systematically talking to the closest access point. This will imply that the access point will be connected between them by a wired infrastructure that will allow the data to travel between acces points and to reach the internet backbone. We consisder that the underlining wired infrastructure is no bottleneck for the transit traffic.

We assume that the mobile users generate a net traffic density of $\mu > 0$ packet per square unit and per slot (excluding retransmissions due to collisions). The aim is to find the appropriate density of access points that permit to serve this traffic.

Theorem 3 *The access point density cannot be smaller than* $\frac{\mu}{\sigma_1}$.

Notice that for a nominal capacity of 10 Mbps, a net traffic density $\mu = 0.5$ (5 Mbps per hectar) would lead to an access point density of that should not be smaller than13.4 access points per hectar (with $\alpha = 2.5$ and $K = 10$).

Proof: Let λ be brute traffic density (including retransmissions). If the access point is at distance r, then the packet will transmitted in average $\frac{1}{p(r,K\lambda)}$ times. Therefore if we assume that the brute traffic density is constant the net traffic density at distance r to the access point will be $p(r, K, \lambda)\lambda$. Since the probability to have the closest access point at distance between r and $r + dr$ is equal to $2\pi\nu_a \exp(-\pi r^2 \nu_a) r dr$ we have the general identity:

$$\mu = \nu_a \int_0^\infty 2\pi\lambda \exp(-\pi r^2 \nu_a) p(r, K, \lambda) r dr \ . \tag{3}$$

By change of variable we have $\mu = \nu_a \Psi(\frac{\nu_a}{\lambda})$, with $\Psi(\theta) = 2\pi \int_0^\infty rp(r, K, 1)e^{-\theta r^2} dr$.

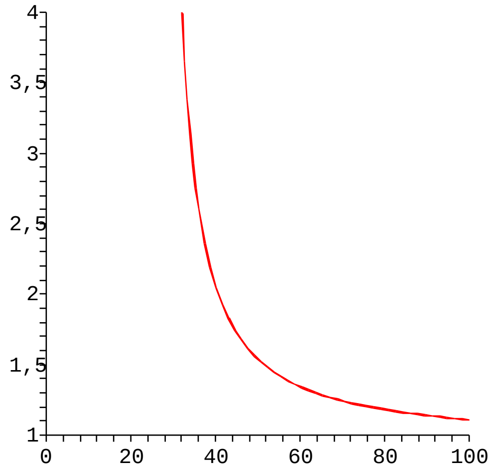

Figure 3: Average number of packet retransmission versus ratio $\frac{\nu_a}{\mu}$ access point density over slot traffic density

Since $\limsup \Psi(\theta) = \sigma_1$ when $\theta \to 0$ we have $\nu_a \geq \frac{\mu}{\sigma_1}$. Notice that when ν is close to $\frac{\mu}{\sigma_1}$ then $\lambda \approx \frac{\Psi'(0)}{(\frac{\mu}{\nu_a} - \sigma_1)\sigma_1}\mu \to \infty$. Notice also that the ratio $\frac{\lambda}{\mu}$ is the average number of packet retransmissions and gives an estimate of the packet delay.

We use the expression $\Psi(y) = \sum_{n>0}(-\frac{K^{-\gamma}}{\Gamma(1-\gamma)})^n \Gamma(1-n\gamma)\frac{\sin(n\pi\gamma)}{n\pi\gamma} y^{n-1}$ to get the exact plot (see figure 3) of parameter $\frac{\lambda}{\mu}$, *i.e.* the average number of packet retransmission as function of relative access point density $\frac{\nu_a}{\mu}$.

4.2 The mesh solution

In this section we investigate the performance of mesh solution when the closest access points may need more than one hop in order to be reached from an arbitrary mobile node.

Theorem 4 *The optimal radius for a single hop transmission is* $\frac{r_0}{\sqrt{\lambda}}$ *where* $r_0 = \arg\max\{rp(r, K, 1)\}$, *leading to an optimal managed neighbor area of* $\sigma_0 = \pi r_0^2$. $\sigma_0 = K^{-\frac{2}{\alpha}}\pi r_1^2$ *where* $r_1 = \arg\max\{rp(r, 1, 1)\}$.

We use the expansion:

$$P(W(\lambda) < x) = \sum_{n\geq 0}(-C\lambda)^n \frac{\sin(\pi n\gamma)}{\pi}\Gamma(n\gamma)x^{-n\gamma}.$$

with $w(\theta) = \exp(-\lambda C\theta^\gamma)$. For $\alpha = 2.5$ we numerically get $r_0 \approx 0.09$, $p_0 = p(r_0, K, 1) \approx 0.75$ and $\sigma_0 = 0.025$.

Proof: The optimal radius is the radius r that minimizes the number of retransmission in order to reach a destination at physical distance L. The number of hops being $\frac{L}{r}$, each hop leading to an average of $\frac{1}{p(r,K,\lambda}$ retransmission, the choice of hop radius r will lead to $\frac{L}{rp(r,K,\lambda)}$. The optimal is attained when $rp(r, K, \lambda)$ is maximal. Notice that $p(r, K, \lambda) = p(K^{\frac{1}{\alpha}} r, 1, \lambda)$, therefore $p_0 = p(r_0, K, 1)$ does not depends on K.

Theorem 5 *If the access point are uniformly distributed with density ν_a then the average hop count h to closest access point is $h = \frac{\sqrt{\lambda}}{2r_0\sqrt{\nu_a}}$. Provided that $h \gg 1$ then the brute traffic density λ and the net traffic density satisfy the identity*

$$2\sqrt{\lambda\nu_a} p_0 r_0 = \mu \tag{4}$$

and $h = \frac{\mu\pi}{2p_0\sigma_0\nu_a}$.

Consequently if $h = 8$, $\mu = 0.5$ per hectar and per slot, then the access point density is 5.23 per hectar, with an average number of 10.6 packet retransmissions.

Proof: We have the identity $\mu = \frac{p_0}{h}\lambda$. On the other hand we have the average distance to the closest access point equal to $\frac{1}{2\sqrt{\nu_a}}$, thus $h = \frac{1}{2r_0}\sqrt{\frac{\lambda}{\nu_a}}$. The condition $h \gg 1$ states that each hop will be close to the optimal $\frac{r_0}{\sqrt{\lambda}}$. Therefore one can express λ and h as a function of (μ, ν_a).

Using Gupta and Kumar results [2] one can state some limits in mesh connectivity. In particular we need to have that each node in the same connected component with its closest access point. If N is the average size of the set of mobile nodes connected to an access point, then the average degree of each mobile node (*i.e.* its neighbor size) must be greater than $\log N$. This impact the net traffic load per user.

Theorem 6 *Let ν be the mobile node density, then the result of theorem 5 is valid when the net traffic density μ is bounded by $\frac{2p_0\sigma_0\sqrt{(\nu+\nu_a)\nu_a}}{\sqrt{\pi \log \frac{\nu+\nu_a}{\nu_a}}}$.*

If $\nu = 100$ nodes per hectar and $\nu_a = \frac{\nu}{10}$ (one access point every 10 users or ten access point per hectar) we get $\mu = 0.44$, *i.e.* 4.4 Mbps per hectar if nominal capacity is 10 Mbps. With access points without mesh this capacity would need more than 12 access points per hectar. For $\nu = 1000$ we get $\mu = 0.98$ still with 10 access points, while this number rises above 26 without mesh.

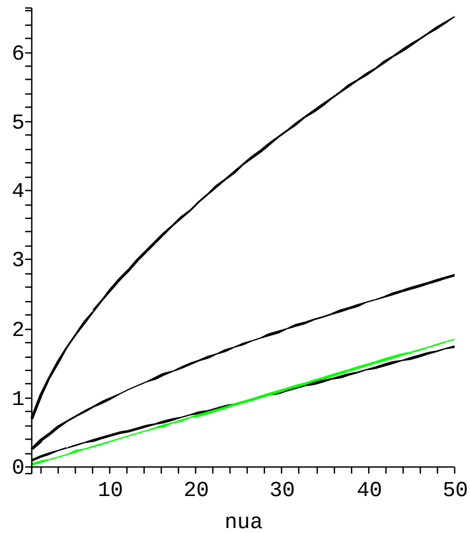

Figure 4: Maximum net traffic capacity μ per hectar and per slot as function of access point density per hectar, without mesh (green) with mesh for various mobile density (black from bottom to top: $\nu = 100$,1,000,10,000 per hectar.

Proof: We have $N = \frac{\nu}{\nu_a} + 1$ and the average neighbor size $M = \frac{\sigma_0}{\lambda}(\nu + \nu_a)$. Therefore we get $\frac{\sigma_0}{\lambda}(\nu + \nu_a) \geq \log \frac{\nu+\nu_a}{\nu_a}$ or $\lambda \leq \frac{\sigma_0(\nu+\nu_a)}{\log \frac{\nu+\nu_a}{\nu_a}}$. Using $\sqrt{\lambda} = \frac{\mu}{2p_0 r_0} \frac{1}{\sqrt{\nu_a}}$ we get the expected result.

When the net traffic demand is above the threshold, then it does not necessarily mean that the network is disconnected. In fact the mesh solution can still hold but the hop radius will exceed the optimal value and the result of theorem 5 would not hold. The determination on how the performance of the network will evolve is for future work.

Figure 4 displays the value of the maximum capacity for different value of access point density and mobile node density. The maximum capacity of the scheme without mesh does not depend on the mobile capacity but is relatively low. With mesh schemes, the maximum capacity increases with mobile node density ν, we display the values for $\nu = 100$ per hectar (semi-urban condition), $\nu = 1,000$ (crowded street, maul or railway station density), $\nu = 10,000$ (stadium density).

References

[1] P. Jacquet, “Elément de théorie analytique de l’information, modélisation et évaluation de performances,” INRIA Research Report RR-3505, http://www.inria.fr/rrrt/rr-3505.html, 1998.

[2] P. Gupta and P.R. Kumar, *The capacity of wireless networks*, IEEE Transactions on Information Theory, vol. IT-46(2), pp. 388-404, March 2000.

[3] P. Jacquet, Control of mobile ad hoc networks, ITW 2006, Uruguay, 2006.

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